



## **CME'21 Virtual event**

**Critical thinking in mathematics: Perspectives and challenges**  
**University of Rzeszow, Rzeszow, POLAND**  
**28-29 JUNE 2021**

### **ABSTRACTS of Research reports**

Monday, 28.06.2021, 14:00-16:00, RR session A

## **ADULTS' KNOWLEDGE OF CHILDREN'S NUMERICAL COMPETENCIES**

Pessia Tsamir<sup>a</sup>, Esther S. Levenson<sup>a</sup>, Dina Tirosh<sup>a</sup>, Ruthi Barkai<sup>a,b</sup>,

<sup>a</sup>Tel Aviv University, <sup>b</sup>Kibbutzim College of Education, Israel

Recognizing that young children engage with numerical activities outside of the school setting, this study investigates adults' knowledge regarding children and numerical activities. Questionnaire were handed out to 92 adults, none of whom were preschool teachers. Questions focused on the numerical competencies adults believed could be promoted during early childhood and the level of difficulty of numerical skills. Findings indicated that most participants mentioned counting, but did not necessarily differentiate between verbal and object counting. Few mentioned skip counting or counting backwards. Adults were aware of specific skills that might be difficult for children to carry out.

# **INTUITION AND REASONING - ANALYSING SECONDARY SCHOOL STUDENTS' CRITICAL THINKING BASED ON PARADOXES AND SOPHISMS**

Mirosława Sajka

Pedagogical University of Krakow, Poland

The topic of the paper is related to the problem of examining and, at the same time, shaping critical thinking in secondary school students. The research was conducted to analyse students' ability to assess the correctness of statements and reasonings, especially when this assessment could be influenced by intuition. With these aims in mind, the research tool was composed using sophisms and paradoxes. Students firstly assessed the correctness of the statements themselves, and were then asked to read a reasoning (true or false), assess the correctness of that reasoning, and reassess the correctness of the statements themselves. The questionnaire survey was a pilot study. A total of 52 people were questioned; the respondents were secondary school students. On the basis of the survey, some preliminary answers were given to several specific research questions. Among them the conclusion is drawn that the reasoning attached to the statements, both paradoxes and sophisms, caused the majority of students to change their assessment of the correctness of the given sentences.

## **DIAGNOSTIC TEACHING AND EDUCATIONAL SUPPORT IN PRESERVICE TEACHER TRAINING**

Sabine Vietz, Tobias Huhmann

University of Education, Weingarten, Germany

Diagnostic teaching and educational support form the basis for adequately addressing heterogeneity in learning mathematics. A new approach for an education module in the second phase of teacher education offers trainee teachers the opportunity to acquire these competencies in a theory-based and practice-oriented way: video-recorded microteaching and accompanying supervision are two elements of the approach, which is designed to encourage critical reflection on teaching performance. At the same time, this approach is intended to enhance the professionalisation in diagnostic teaching and supporting. The results of the module-evaluation show that these specific elements encourage and foster critical reflection, and this is seen essential to develop professionalisation.

# **WORD PROBLEMS DEVELOPING CRITICAL THINKING OF PUPILS AS SEEN BY PRIMARY SCHOOL PROSPECTIVE TEACHERS**

Eva Nováková

Faculty of Education, Masaryk University, Brno, Czech Republic

The paper presents partial outcomes of a research aimed at analysing students' approaches to selected non-standard mathematical tasks. We studied ways of students' analysis and reflection of tasks, which – as we assume – have some potential to develop critical thinking of primary school pupils. We believe that this competence is a factor shaping professional identity of prospective teachers as reflective people of practice. Our findings suggest that the topic discussed in our contribution can be used in a wide range of activities in real-life education practice.

Monday, 28.06.2021, 14:00-15:30, RR session B

## **KNOWLEDGE AND SELF-EFFICACY OF MATHEMATICS TEACHERS IN SPECIAL EDUCATION CLASSES FOR LEARNING-DISABLED STUDENTS: THE DIFFERENCES BETWEEN MULTIPLICATION AND DIVISION**

Rachel Filo & Iris Schreiber

Kibbutzim college of education, Israel

This article describes a study investigating teaching multiplication and division in special education classes for learning disabled students. The study explored 64 teachers regarding two factors which affect teaching-learning processes: teachers' knowledge (common content knowledge, specialized content knowledge, knowledge of content and teaching, knowledge of content and students) and their self-efficacy regarding their knowledge. The findings indicated that the teachers have greater knowledge and higher self-efficacy regarding multiplication than they have regarding division.

# **MANIFESTATIONS OF CRITICAL THINKING IN THE PROCESS OF SOLVING TASKS BY SEVENTH GRADERS**

Edyta Juskowiak

Adam Mickiewicz University in Poznan, Poznan, Poland

The article contains information about the preliminary results of the research conducted in a group of 14-year old students from four classes. The main research aims to check students' readiness to use formal operations to solve mathematical problems in the field of geometry. In addition, these solutions were analyzed for the characteristics of critical thinking.

## **ANTINOMIES OF PROBLEM POSING**

Zoltán Kovács, Eszter Kónya

University of Nyíregyháza and University of Debrecen, Hungary

This paper is motivated by the question, “What does a classroom look like when students engage in problem-posing activities?” In particular, “What are the key features of effective problem posing and problem-posing instruction in classrooms?” The qualitative research is based on the investigation of problem-posing episodes for 5-8<sup>th</sup> graders. The authors investigate some phenomena accompanying the implementation of problem-posing activities and formulate them as antinomies. They also make suggestions for resolving these antinomies

Tuesday, 29.06.2021, 11:00-13:00, RR session A

## **TOOL-TASK DIALECTIC IN MATHEMATICS CLASSROOMS**

Huey Lei

University of Saint Joseph, Macao SAR, China

Education innovation is a distinct trend in education research, with the use of manipulatives a key research field, embracing pedagogical development in mathematics education. Tool-based pedagogy involving concrete and technological tools in mathematics classrooms helps create an interactive environment for the construction of students' mathematics knowledge, through the appropriate use of tools and the orchestration of mathematics teachers. These tools support students' abilities to construct cognitive models while harmonising

contexts of the gestures made and language used by them. This empirical research proposes a new idea of Tool-Task dialectic, grounded in the analysis of mathematics lessons with designed tool-based tasks.

## **ENCOURAGING DISCOVERY IN SUBSTANTIAL LEARNING ENVIRONMENTS: DESIGNING PLAY AND DOCU ROOMS**

Tobias Huhmann, Ellen Komm

University of Education, Weingarten, Germany

Despite the established status of discovery learning as a fundamental teaching principle it is still lagging behind in everyday teaching. The concept substantial learning environment offers opportunities to constructively respond. After years of accompanying teacher-students in practical teaching we have developed a concept to design the so called “play and docu(menting) rooms” within substantial learning environments. Our objective is to support students in processes and products of discovering by using their own documentations. In this context, questions rise about the conditions for successfully designing documentation and dynamization processes that support individual learning as well as questions about the ways of using such a designed learning environment.

## **TEACHING STRATEGIES FOR DEVELOPING CRITICAL THINKING SKILLS**

Emőke Báró

University of Debrecen, Hungary

In recent years, it has been shown in several studies that pupils’ interest in mathematics has been decreasing. As a math teacher, the author feels it is her responsibility to change something about this situation. The first steps to achieve this aim are based on holding some problem-based activities. In this paper, the author will discuss teaching methods as well as problem-solving strategies concerning two different activities. Pupils were solving problems using different heuristic methods, such as working backwards and pattern recognition. The observations on the classes were analyzed by different factors, such as cooperative learning, mathematical game, problem posing, and difficulties of teaching.

# **COMPARATIVE ANALYSIS OF TEXTBOOKS AS A WAY TO DEVELOP CRITICAL THINKING IN MATHEMATICS TEACHERS**

Barbara Pieronkiewicz, Małgorzata Zambrowska

Pedagogical University of Cracow, The Maria Grzegorzewska University,  
Poland

Researchers agree that one of the main objectives of education is to develop students' ability to think critically. To support the development of such a skill in their students, teachers must first become critical thinkers themselves. In this article we elaborate on the possibility of developing critical thinking skills of pre-service mathematics teachers through the means of comparative analysis of textbooks. We illustrate the potential of this activity providing a review of different textbook treatments of the common fractions division.

Tuesday, 29.06.2021, 11:00-13:00, RR session B

## **MATHEMATICS IN THE KINDERGARTEN: CONTINUING AND COMPLETING A REPEATING PATTERN**

Iris Schreiber

Kibbutzim College of Education & Bar Ilan University, Israel

This paper reports a study on the patterning knowledge of 206 Israeli children, aged 4–6. The children were given a repeating pattern (consisting of circles and squares) and were asked to continue and to complete it. For each task the children were given a set of shapes from which they chose the appropriate ones for performing the task. Some sets include exactly the necessary shapes, some sets lack the required shapes, whereas others include surplus shapes. The results show that the given set affects children's performance.

# **THE ROLE OF CRITICAL THINKING IN DATA-BASED ARGUMENTATION – EMPIRICAL FINDINGS FROM STUDIES WITH PRIMARY STUDENTS**

Jens Krummenauer, Sebastian Kuntze

Ludwigsburg University of Education, Germany

For argumentation based on statistical data, elements of critical thinking can be expected to be helpful, e.g., when questioning others' interpretations of data or for developing counter-arguments. However, empirical evidence in this regard is scarce, especially as far as young learners are concerned. In a re-analysis of both qualitative and quantitative data from prior studies with primary students, we investigated what role critical thinking plays in students' data-based argumentations. The results indicate that elements of critical thinking can play both a supportive, but also a non-supportive role.

## **DO STUDENTS ANALYZE AND EVALUATE THE RESULT OF THEIR PROBLEM SOLVING ACTIVITY?**

Márton Kiss, Eszter Kónya

University of Debrecen, Hungary

In this paper, we investigate 9<sup>th</sup> grade students work while they solve mathematical problems where more possible cases or even an impossible case appear. We planned a developing experiment which aimed at accustoming students to handle a mathematical problem in a conscious way and from a wider perspective. Furthermore, another aim was to change the inaccurate belief that each mathematical problem has an answer and that is the only answer.

## **PROBABILITY KNOWLEDGE EFFECT ON CRITICAL THINKING IN YOUNG AGES**

Zorzos Michail, Avgerinos Evgenios

University of the Aegean, Greece

Critical thinking is an indispensable advantage for any rationally thinking citizen (Aizikovitsh & Amit, 2008). Mathematics is a characteristic domain of science, that promotes the development of critical thinking. Specifically, Probabilities, because of their nature and their impact to everyday life, may be the domain of mathematics that can directly benefit critical thinking processes. This paper investigates the relation between critical ability and probability thinking. The

research was conducted on primary school students. The results show that students appear to improve their critical skills after a quick probability lesson.

Tuesday, 29.06.2021, 14:00-15:30, RR session A

## **PROBLEM SOLVING: HOW DO STUDENTS WITH DIFFERENT PERSONALITY TYPES SHOW THEIR CRITICAL THINKING WHEN SOLVING A MATHEMATICAL PROBLEM?**

Linda Devi Fitriana

University of Debrecen, Hungary

Personality becomes a factor which indirectly influence student critical thinking. This research aims to investigate how students with different personality types show their critical thinking when solving a mathematical problem, as identified by the application of Keirsey's theory. The subjects are junior secondary school students who represent each personality type by Keirsey. Qualitative data were collected through a problem solving test and interview. The result underlines that all students propose different conclusions for the problem and show their critical thinking according to the nature inherent in their personality type. This condition can be a beneficial input for teachers in designing instruction for preparing students to be better in critical thinking by considering their character.

## **CRITICAL THINKING OF STUDENTS IN THE PROCESS OF GENERALIZATION**

Anna Pyzara

Maria Curie-Skłodowska University, Lublin, Poland

Every teacher should have critical thinking skills. This research aims to examine whether prospective teachers of mathematics use critical thinking. To this end, I analysed their activities related to the generalization process. The results of my study show that students used critical thinking in their work, but not enough. Students were able to follow a pattern and recognize the rule of conduct in simple situations, but they had difficulties with the description and justification of the rule, especially when it required the consideration of different cases.



# **DEVELOPMENT OF ABSTRACT THINKING THROUGH HANDS-ON ACTIVITIES AND ALGEBRAIC MODELS**

Ivona Grzegorzcyk

California State University Channel Island, USA

In this study pre-service teachers had to modify their lessons to include interactive activities and algebraic modelling. We analyse the performance of the teachers' learning and pedagogy changes. We also evaluate performance and attitudes of sixth-graders and ninth-graders participating in the interactive lessons presented by the teachers. All participants supported hands-on modelling to routine lecture-based teaching.

Tuesday, 29.06.2021, 14:00-15:00, RR session B

## **DOES THIS EQUATION DESCRIBE THIS SITUATION? EXPLORING THE ALGEBRAIC THINKING OF ELEMENTARY STUDENTS**

Esperanza López Centella<sup>a</sup>, Jana Slezáková<sup>b</sup>, Darina Jirotková<sup>b</sup>

<sup>a</sup> University of Granada, Spain; <sup>b</sup> Charles University, Czech Republic

In the present study we explore qualitatively the conceptual and semantic understanding of equations of elementary school pupils who were taught how to solve basic linear equations. We analyse the individual answers of 38 grade five students in a public primary school to a paper-based task consisting in justifying whether the equation  $5+2x=13$  models each one of 5 contextualized situations. Under a grounded theory approach, we provide a system of categories of the students' strategies. In particular, our findings show the abilities of the students to deal with the task and to infer true mathematical facts about equations, the variety of the students' strategies and the dependency of the strategy on the way in which the situation is presented. Teaching implications are derived.

## **STUDENTS' CREATIVE THINKING DURING SOLVING ALGEBRAIC TASKS**

Marta Pytlak

University of Rzeszow, Poland

Mathematics education is not only about teaching concepts, theorems and algorithms. It is also teaching the skills which students need in everyday life.

One of them is reflective and critical thinking. It is a skill that can be successfully developed during math lessons. This kind of thinking supports many mathematical activities, such as discovering regularities and generalizations. The paper presents some results of research concerning algebraic thinking and generalization. We are trying to answer the question: does reflective thinking support the ability of generalizing and perceiving regularity.